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AS Level 9609

MATHEMATICS

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ARSLAN TANVIR SHEIKH

03214525069

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Prepared by:	Arslan Tavir Sh.
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Content

1. 9709/11/m/j/13, Q8(i)

(i) Express $2x^2 - 12x + 13$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

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2. 9709/12/m/j/13, Q#3

The straight line $y = mx + 14$ is a tangent to the curve $y = \frac{12}{x} + 2$ at the point P. Find the value of the constant m and the coordinates of P. [5]

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3. 9709/12/o/n/13, Q10(ii),(iii)

A curve has equation $y = 2x^2 - 3x$.

(ii) Express $2x^2 - 3x$ in the form $a(x+b)^2 + c$, where a, b and c are constants, and state the coordinates of the vertex of the curve. [4]

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The functions f and g are defined for all real values of x by

$$\mathrm{f}(x) = 2x^2 - 3x \quad \text{and} \quad \mathrm{g}(x) = 3x + k,$$

where k is a constant.

(iii) Find the value of k for which the equation $\mathrm{gf}(x) = 0$ has equal roots. [3]

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4. 9709/13/o/n/13, Q#1

Solve the inequality $x - 2x - 2 > 0$. [3]

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5. 9709/11/m/j/14, Q#2,Q#11(i)

(i) Express $4x^2 - 12x$ in the form $(2x + a)^2 + b$. [2]

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(ii) Hence, or otherwise, find the set of values of x satisfying $4x^2 - 12x > 7$. [2]

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6. A line has equation $y = 2x + c$ and a curve has equation $y = 8 - 2x - x^2$.

(i) For the case where the line is a tangent to the curve, find the value of the constant c. [3]

7. 9709/12/m/j/14, Q#10(iii),(iv)

Functions f and g are defined by

$$f : x \mapsto 2x - 3, \quad x \in \mathbb{R},$$
$$g : x \mapsto x^2 + 4x, \quad x \in \mathbb{R}.$$

(iii) Find the set of values of x for which $g(x) > 12$. [3]

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(iv) Find the value of the constant p for which the equation $\mathrm{gf}(x) = p$ has two equal roots. [3]

8. 9709/13/m/j/14, Q#8,Q#9(i)

(i) Express $2x^2 - 10x + 8$ in the form $a(x + b)^2 + c$, where a, b and c are constants, and use your answer to state the minimum value of $2x^2 - 10x + 8$. [4]

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(ii) Find the set of values of k for which the equation $2x^2 - 10x + 8 = kx$ has no real roots. [4]

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9. The base of a cuboid has sides of length x cm and $3x$ cm. The volume of the cuboid is 288 cm^3.

(i) Show that the total surface area of the cuboid, A cm^2, is given by

$$A = 6x^2 + \frac{768}{x}.$$ [3]

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10. 9709/11/o/n/14, Q#5,Q#10(i),(iii)

Find the set of values of k for which the line $y = 2x - k$ meets the curve $y = x^2 + kx - 2$ at two distinct points. [5]

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11. **(i)** Express $x^2 - 2x - 15$ in the form $(x + a)^2 + .b$ [2]

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The function f is defined for $p \leqslant x \leqslant q$, where p and q are positive constants, by

$$f : x \mapsto x^2 - 2x - 15.$$

The range of f is given by $c \leqslant f(x) \leqslant d$, where c and d are constants.

For the case where $c = 9$ and $d = 65$

(iii) find p and q, [4]

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12. 9709/12/o/n/14, Q#6(i)

The equation of a curve is $y = x^3 + ax^2 + bx$, where a and b are constants.

(i) In the case where the curve has no stationary point, show that $a^2 < 3b$. [3]

13. 9709/13/o/n/14, Q#3(i),Q#9(i)

(i) Express $9x^2 - 12x + 5$ in the form $(ax + b)^2 + c$. [3]

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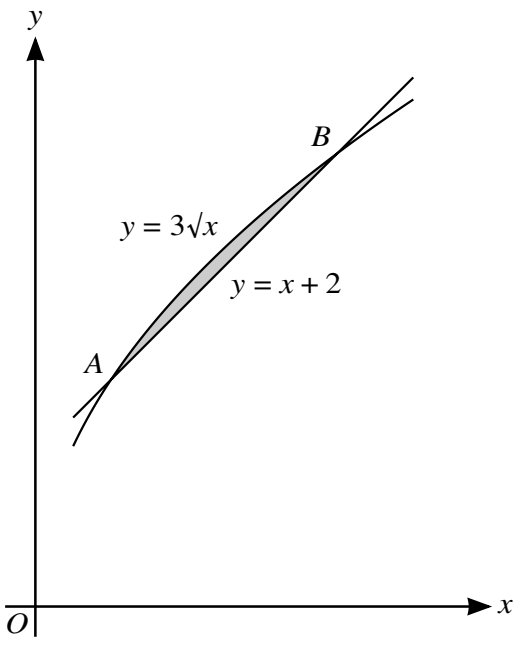
14.



The diagram shows parts of the graphs of $y = x + 2$ and $y = 3\sqrt{x}$ intersecting at points A and B.

(i) Write down an equation satisfied by the x-coordinates of A and B. Solve this equation and hence find the coordinates of A and B. [4]

15. 9709/11/m/j/15,Q#9(iii),Q#5(ii)

The equation of a curve is $y = x^3 + px^2$, where p is a positive constant.

Another curve has equation $y = x^3 + px^2 + px$.

(iii) Find the set of values of p for which this curve has no stationary points. [3]

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16. **(ii)** Express A in the form $a - (r - b)^2$, where a and b are constants. $A = 12r - r^2$ [2]

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17. 9709/12/m/j/15,Q#11(i),(ii)

The function f is defined by $f : x \mapsto 2x^2 - 6x + 5$ for $x \in \mathbb{R}$.

(i) Find the set of values of p for which the equation $f(x) = p$ has no real roots. [3]

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The function g is defined by $g : x \mapsto 2x^2 - 6x + 5$ for $0 \leqslant x \leqslant 4$.

(ii) Express $g(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

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18. 9709/13/m/j/15,Q#1

Express $2x^2 - 12x + 7$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

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19. 9709/11/o/n/15,Q#3,Q#6

Solve the equation $\sin^{-1}(4x^4 + x^2) = \frac{1}{6}\pi$. [4]

20. A curve has equation $y = x^2 - x + 3$ and a line has equation $y = 3x + a$, where a is a constant.

(i) Show that the x-coordinates of the points of intersection of the line and the curve are given by the equation $x^2 - 4x + (3 - a) = 0$. [1]

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(ii) For the case where the line intersects the curve at two points, it is given that the x-coordinate of one of the points of intersection is -1. Find the x-coordinate of the other point of intersection. [2]

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(iii) For the case where the line is a tangent to the curve at a point P, find the value of a and the coordinates of P. [4]

21. 9709/12/o/n/15,Q#8

The function f is defined, for $x \in \mathbb{R}$, by $f : x \mapsto x^2 + ax + b$, where a and b are constants.

(i) In the case where $a = 6$ and $b = -8$, find the range of f. [3]

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(ii) In the case where $a = 5$, the roots of the equation $f(x) = 0$ are k and $-2k$, where k is a constant. Find the values of b and k. [3]

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(iii) Show that if the equation $f(x + a) = a$ has no real roots, then $a^2 < 4(b - a)$. [3]

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22. 9709/13/o/n/15,Q#1,Q#3(i)

A line has equation $y = 2x - 7$ and a curve has equation $y = x^2 - 4x + c$, where c is a constant. Find the set of possible values of c for which the line does not intersect the curve. [3]

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23. **(i)** Express $3x^2 - 6x + 2$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

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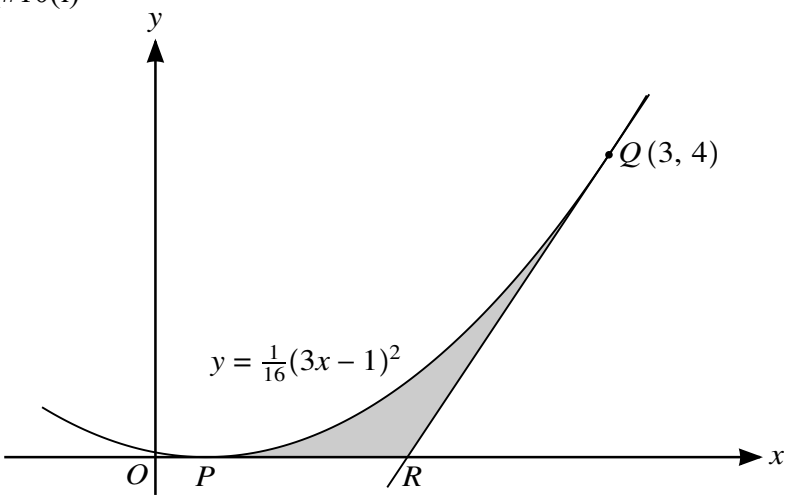
24. 9709/12/f/m/16,Q#10(i)



The diagram shows part of the curve $y = \frac{1}{16}(3x - 1)^2$, which touches the x-axis at the point P. The point $Q\,(3, 4)$ lies on the curve and the tangent to the curve at Q crosses the x-axis at R.

(i) State the x-coordinate of P. [1]

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25. 9709/11/m/j/16,Q#6(a),Q#6(b)(ii)

(a) Find the values of the constant m for which the line $y = mx$ is a tangent to the curve $y = 2x^2 - 4x + 8$. [3]

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(b) The function f is defined for $x \in \mathbb{R}$ by $f(x) = x^2 + ax + b$, where a and b are constants. The solutions of the equation $f(x) = 0$ are $x = 1$ and $x = 9$. Find

(ii) the coordinates of the vertex of the curve $y = f(x)$. [2]

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26. 9709/12/m/j/16,Q#11(i),(ii),(iii),(iv)

The function f is defined by $f : x \mapsto 6x - x^2 - 5$ for $x \in \mathbb{R}$.

(i) Find the set of values of x for which $f(x) \leqslant 3$. [3]

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(ii) Given that the line $y = mx + c$ is a tangent to the curve $y = f(x)$, show that $4c = m^2 - 12m + 16$. [3]

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The function g is defined by g : $x \mapsto 6x - x^2 - 5$ for $x \geqslant k$, where k is a constant.

(iii) Express $6x - x^2 - 5$ in the form $a - (x - b)^2$, where a and b are constants. [2]

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(iv) State the smallest value of k for which g has an inverse. [1]

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